

Operations Technical Services - Quantitative Methods

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Agenda

- QM Center of Excellence
- Why We Do It
- Role
- Statistical Modeling
- Success Story
- Frequent Questions



QM Center of Excellence (CoE)

- Personnel
 - 2019 – 1 chief, 1 statistician
 - **2021** – 1 chief, 2 statisticians, 4 QM technical specialists
 - 2024 – 1 chief, 4 statisticians, 9 QM technical specialists
- Mission – Establish and implement the gold standard for statistical expertise and practice in DCAA and the DoD
- Program Areas
 - Statistical Sampling (Variable and Attribute)
 - Statistical Modeling (Graphic and Regression Analysis, Improvement Curves)

We are here to support the warfighter by supporting you!

What does that mean?

- QM CoE assists if
 - You have data for an audit and aren't sure what to do with it
 - You have audit data and know what to do with it but aren't comfortable with using the technology available to do it
 - You know what to do with the data and how to implement procedures tech-wise but want confirmation it's correct
 - Your “what you want to do” mandates a touchpoint with QM CoE
 - You want assistance translating the statistical jargon into human form for work papers, out-briefs, etc.
 - You want support with other statistical projects

Why do we do it and what do we gain?

- Ensure latest guidance and best practices are implemented
- Ensure statistical validity and defendability of results
- Allow DCAA's expert auditors to focus on audit-specific tasks instead of statistical entanglements (and increase audit efficiency)
- Aid audit teams in:
 - Developing negotiating positions
 - Applying statistics to complete their audits
 - Projecting results when appropriate
 - Explaining statistical processes/results to customers



What does this mean for the customer?

Results that:

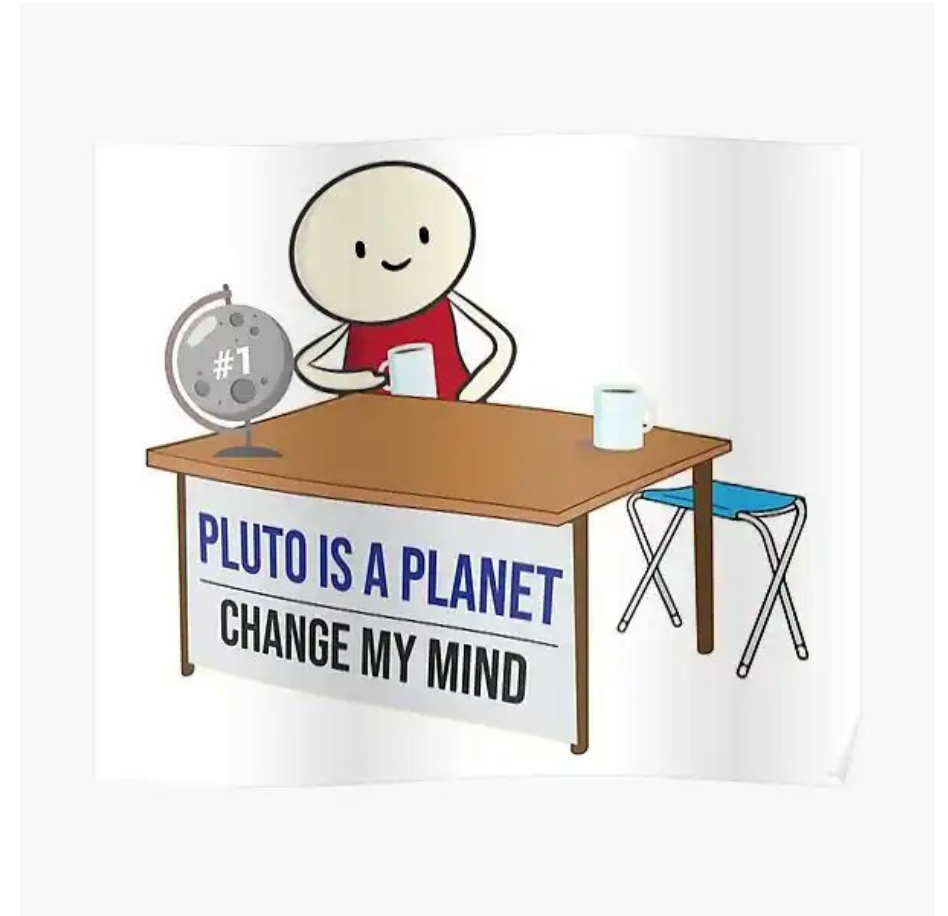
- Neither overstate nor understate what the data tells us
- Are more robust for negotiations
- Are more defensible in litigation

Statistical Modeling

- Statistical modeling uses assumptions about historical data to make statistical inferences (draw conclusions).
- Routine modeling options include:
 - Simple Linear Regression
 - Learning/Improvement Curves
 - Quantity Price Discount Curves (QPDC)
 - Time Series

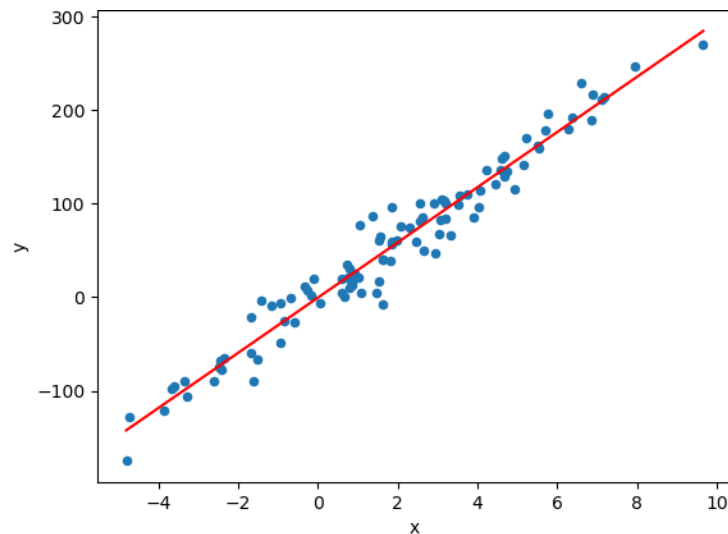
Statistical Modeling

- Making a model to *approximate* reality.
- Imagine your 4th grade model of the solar system. (Was it 8 planets or 9?)
- In statistical modeling, we want to make sure our approximation is as close as possible to reality.

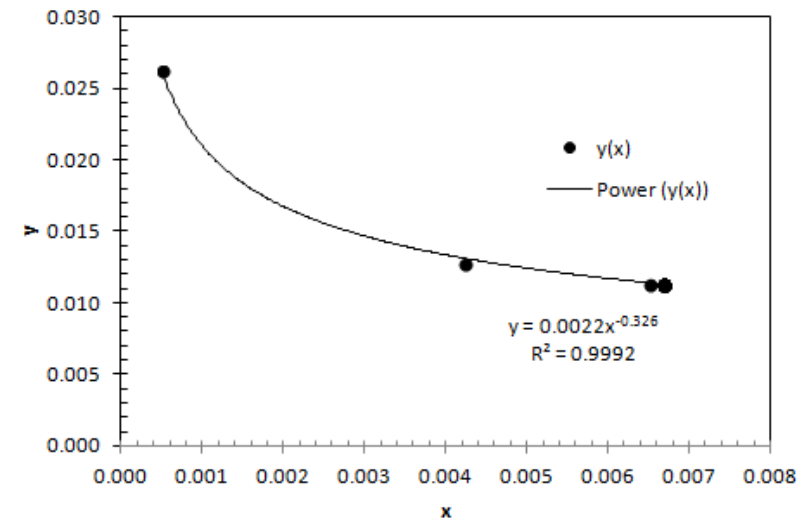


Regression

- Regression is a type of modeling that creates an algebraic equation to describe the relationship between the variables.
- DCAA commonly uses 2 regression types:



Linear Regression



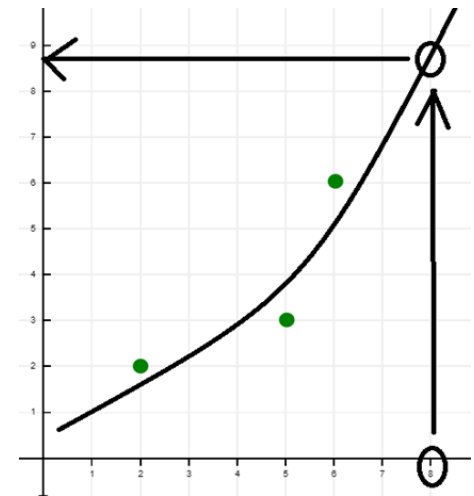
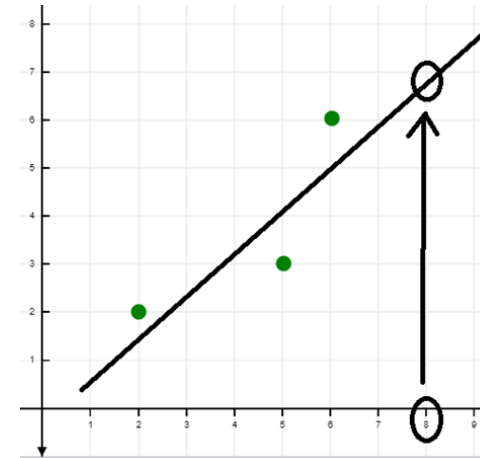
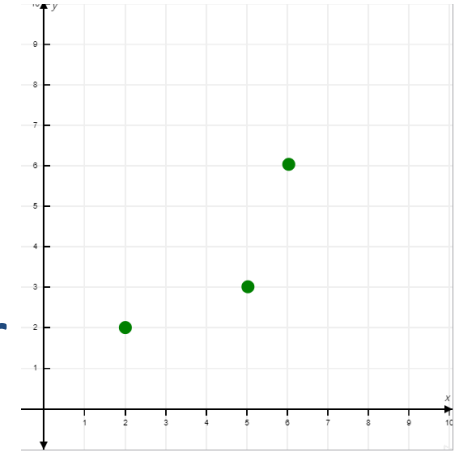
Power Regression

Simple Linear Regression

- Simple linear regression is used when we believe the data's relationship can be modeled by a straight line (plus uncertainty). (e.g. lemonade stand profits)
- “Simple” linear regression is the technical term when we only have one predictor variable.
- If we have more predictor variables, we'd say “multivariate” linear regression or “multiple” linear regression.
- It's called “linear” simply because the model is a straight line.
 - However, if the actual underlying reality was a curve, for example, then when we made a prediction, we could be way off.

Regression Lines

- When you make a regression line, think about it as “connecting the dots,” but with a highlighter and not a fine point pen.
 - The wide line we draw through the points accounts for uncertainty in the data.
- Consider these three graphs.
- Which of the bottom two best portrays the relationship of the data in the top one?



To Use Simple Linear Regression

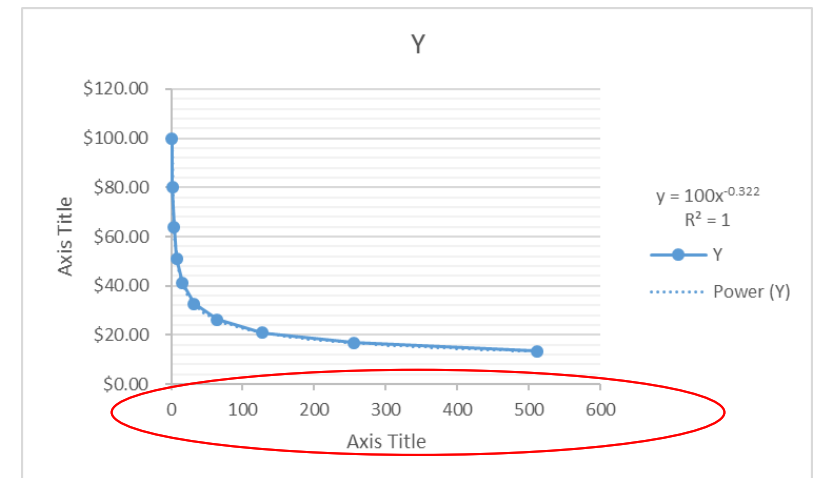
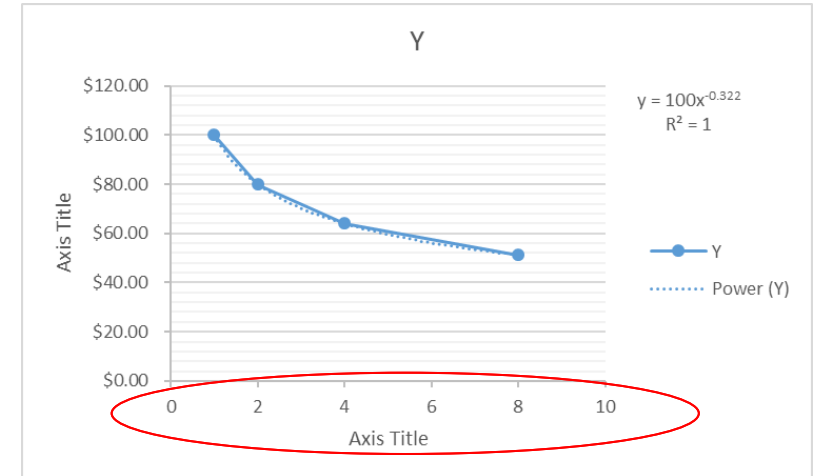
- Must validate several assumptions:
 - Linearity
 - Independence
 - Equality of variances
 - Normality
- These are four normal (i.e. “found in a textbook”) steps that help ensure that our model accurately represents reality.
- QM CoE can help with conducting validations and performing the regression analysis.

Common Power Regressions

- Improvement/Learning Curves
 - Demonstrates learning over time
 - Improvement driven by **cumulative** units produced over time
 - Lots sizes of the same size cost different amounts
 - Tends to be more human involvement
 - Costs expected in general to go down over time
- Quantity Price Discount Curves
 - No expectation of learning over time.
 - Improvement driven by the **quantity** in a lot
 - Lot sizes of the same size cost the same amount
 - Tends to be less human involvement

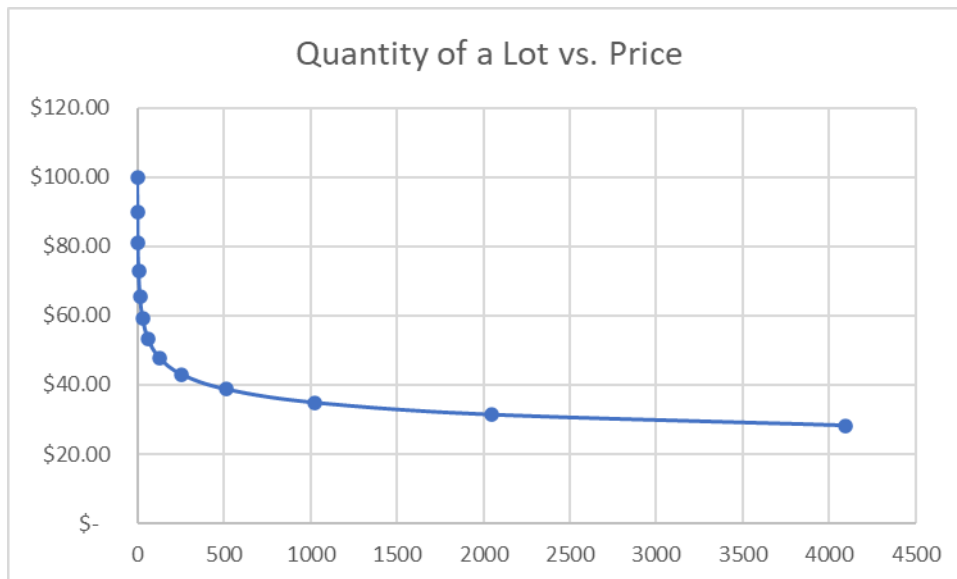
Improvement Curve Notes

- You'll notice on the graph that the slope gets flatter and flatter over time. (example: DIY projects)
- This means that the greatest improvements in efficiency or cost will happen early in the process.
- It also means that as you get far into the process, there will be little improvement.



What is a QPDC?

- QPDC = Quantity Price Discount Curve
- This basic idea is that as more of something is purchased, the price goes down (usually a *curved* scale).



- Note: “quantity” is “quantity produced in a particular lot” and not “total produced over time”
- Also note the price curve gets less and less steep over time

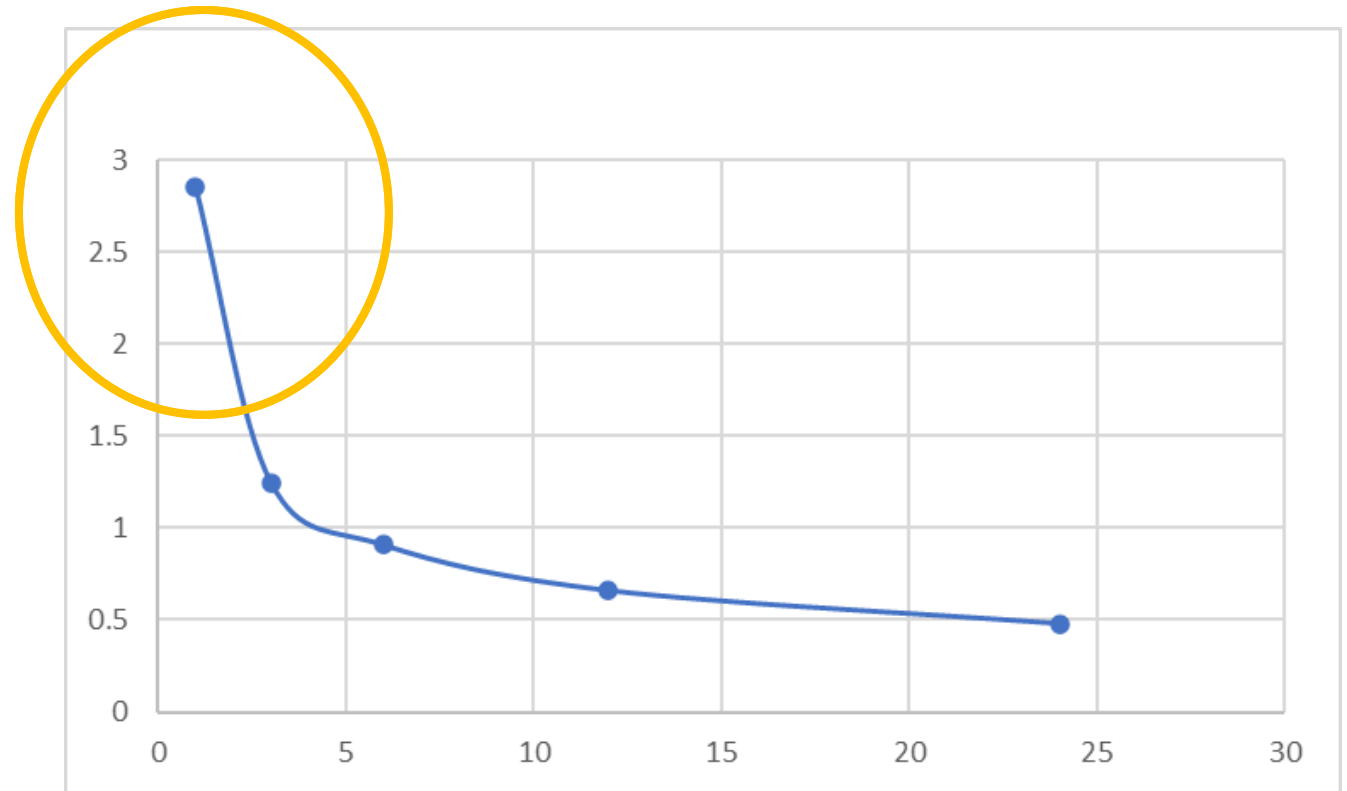
When does a QPDC Work Best?

- When there is a gain in efficiency as more of a product are produced and thus there is a lower production cost at higher quantities. (Think about how making 48 cookies is cheaper per cookie than making 12.)
- When there is good historical evidence of such a relationship and enough data to provide a precise estimate for prices at new quantities.
- When the “price” is in terms of hours.
 - Price in dollars is much harder to standardize.



ABC (Always Be Cautious)

- As an example, think about buying cans of Pepsi.
- Even though we can see there is a price quantity curve, we need to be cautious when applying it.
- For example, here, the curve slope would give us a price of almost \$3 a can if we only bought one can.
- A contractor may actually be purchasing an item in bulk but only list a small quantity in a given purchase order.
- Think about the difference between what a restaurant pays for cans of pop vs. what they charge you for purchasing one with a meal.
- We have added a Power BI tool that lets us analyze lots of parts at once, provided sufficient history is available.



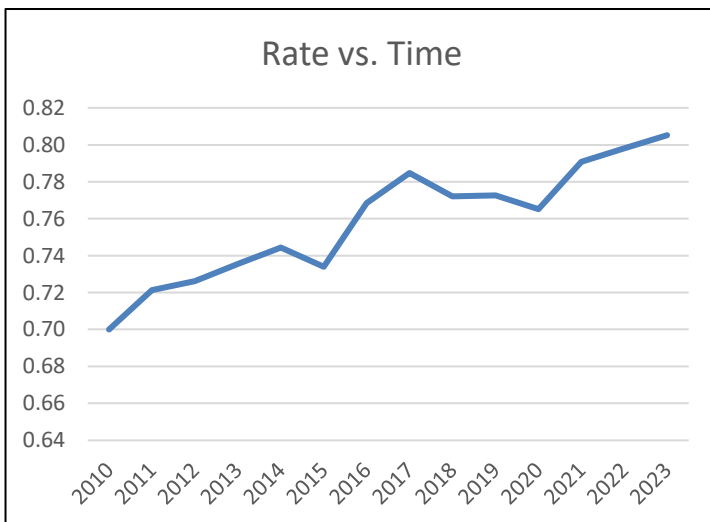
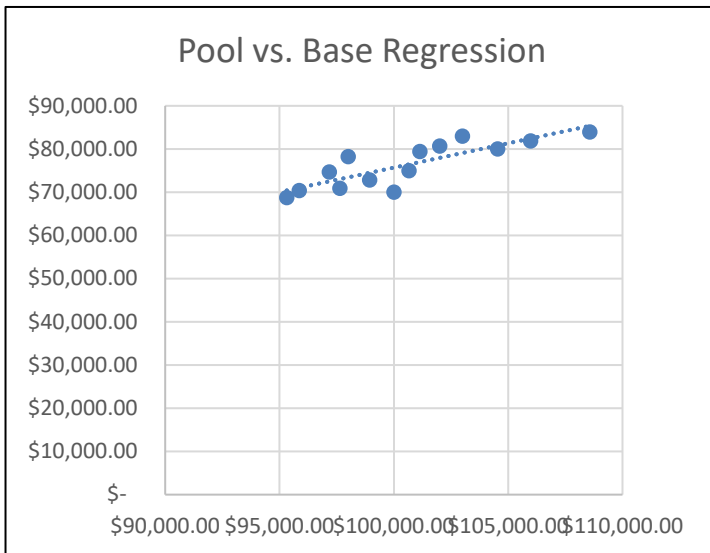
What is Time Series?

- A time series is a statistical model that forecasts a future value for a variable based on the time-ordered historical values of that variable.
 - For example, predicting temperature based on the temperatures from previous days.
 - Another example might be predicting your credit card statement amount based on your historical statements (e.g. summer vacation or holiday spending vice other months).

How are time series different from regression?

- A regression models the relationship of two variables: a Y-variable against a correlated X-variable. (Pool vs. Base, for example)
- A time series models the relationship of only one variable vs. time. (Pool vs. Time, Base vs. Time, or Rate vs. Time, for example)
- **In a sense, a regression assumes the relationship stays the same over time; a time series assumes it might not.**

Regression vs. Time Series



- Both seem to go up and to the right. Isn't that the same thing? No:
 - In the regression, we're just showing that as X gets bigger, Y also gets bigger. We're not exploring if they're getting bigger at the same rate over time.
 - In the time series, we're showing the Y is getting bigger faster than X.
- Why doesn't the regression show this?
 - Because the data is ordered from small to big, not early to later.
 - This masks the change in rate over time.
 - This is why it's important to consider several visualizations instead of running to one or the other models.
- Which one is the "better" model?
 - Here, because there is a clear relationship between the data and time, a time series is probably the right call.
 - (With the understanding that there's always a chance the rate is just randomly going up and a larger data set would show no long-term trend.)

Time Series: Additional Considerations

- The GIGO (garbage in = garbage out) principle applies to time series, as well. If you feed it 4 data points that are unrepresentative of the actual history, you're going to get bad results.
- If possible, you'd have a robust data set (at least 10 years of annual, or 36 months, or 12 quarters).
 - Smaller data sets can be used, but there's extra unquantifiable uncertainty due to that.
- You want to feed the model a whole year's worth of data (so if you have 40 months, only use 36).
- Sometimes a time series produces a prediction interval (similar to a confidence interval) that is so wide that even if the proposed is within, it's not strong evidence of reasonableness.

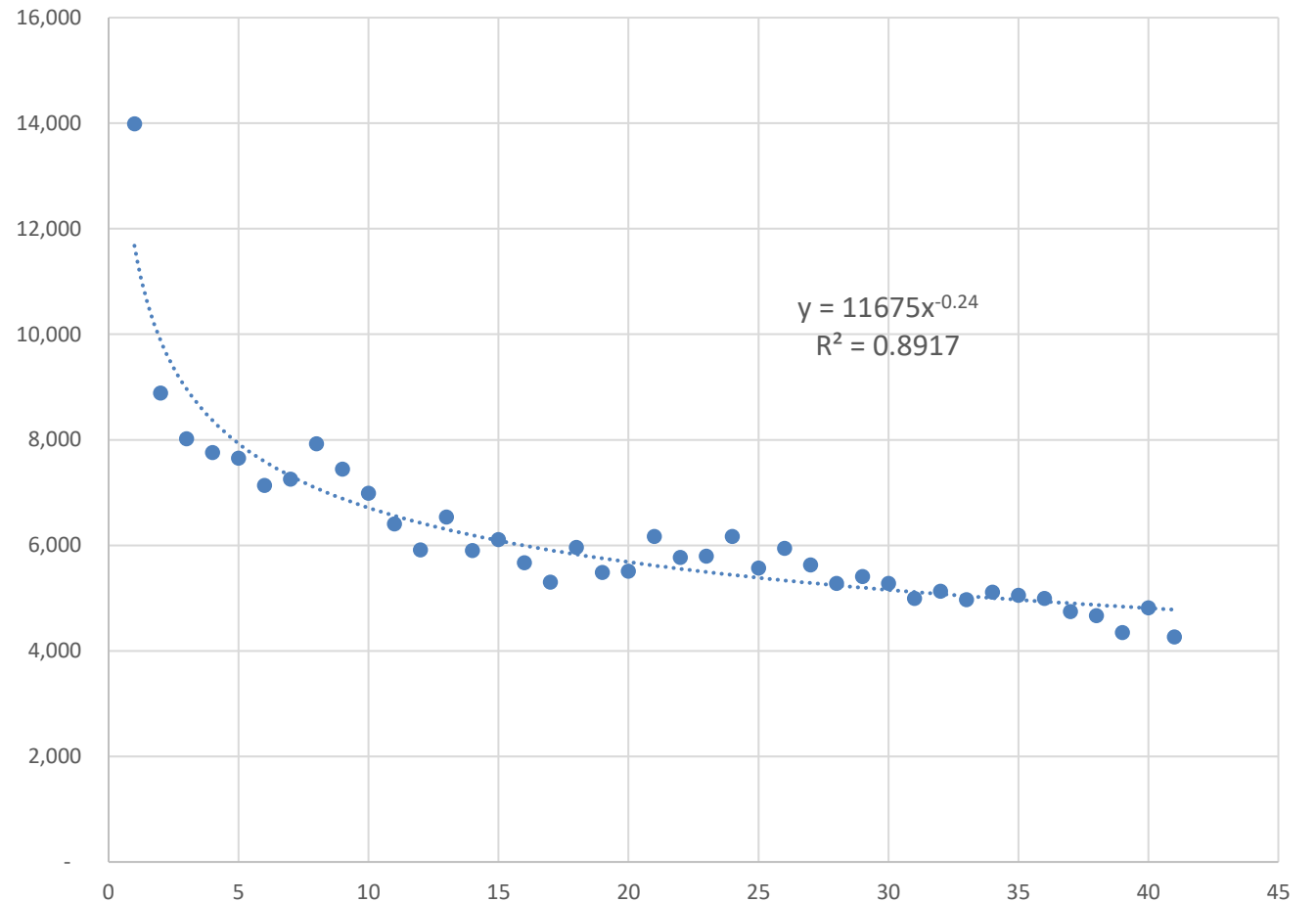
Success Story

- Improvement curve scenario
- Basis of Estimate:

The contractor proposed labor hours are based on historical actual labor hours from a similar program with the application of a learning curve to batches of units. The remainder of the estimate was held fixed as the contractor asserted those hours were not subject to learning.

Improvement Curve Scenario - QM Assistance

- Evaluate contractor-proposed slope
- Analyze historical data to determine relevance
- Determine if an improvement curve is feasible



Improvement Curve Scenario Results

- QM CoE evaluated the historical data and developed a slope more accurate than the contractor proposed.
- Audit team and QM CoE collaborated to understand the nature of the differences. (curve should be applied to full rate production and not just LRIP as proposed)
- Questioned ~ 60K (7%) of proposed touch labor hours and ~ 450K hours when extrapolated to other cost areas (e.g. bulk support)

Frequently Asked Questions

- How many data points do we need?
- Why do we need different amounts of data points for different methods?
- Can you sit in when we discuss additional questions, results, or working papers with the contractor or audit team?
- It doesn't look like regression will work. What else can we try?
- What if we get bad results?
- For requests or questions: dcaa.belvoir.hq.list.qm@mail.mil

Questions?